

## FURTHER ASPECTS OF DROPWISE CONDENSATION THEORY

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**Abstract**—The theoretical dependence of heat-transfer coefficient on maximum drop size [1] is incorporated in a new calculation giving the dependence of sweeping frequency and heat-transfer coefficient on distance down the condensing surface. Both results are in satisfactory agreement with earlier observations. The height-dependence of the heat-transfer coefficient is shown to be extremely weak. It is also shown that the nucleation site density is deducible from an equation for the distribution of drop sizes [1] and that the values obtained are in satisfactory agreement with optical and electron microscope measurements.

### NOMENCLATURE

<p><math>b_z</math>, width of swept track at distance <math>z</math> from top of surface;</p> <p><math>b_0</math>, width of swept track at <math>z = z_0</math>;</p> <p><math>c_1, c_2, c_3</math>, } quantities independent of <math>z</math> [see equations (8), (11) and (13)];</p> <p><math>g</math>, local gravitational acceleration;</p> <p><math>h</math>, heat-transfer coefficient;</p> <p><math>h_z</math>, heat-transfer coefficient at distance <math>z</math> from top of surface;</p> <p><math>h_0</math>, heat-transfer coefficient at <math>z = z_0</math>;</p> <p><math>h_{fg}</math>, specific enthalpy of vapour-liquid phase change;</p> <p><math>K_1</math>, a constant;</p> <p><math>K_2</math>, <math>K_2^* + l_p \dot{Q}_2'' / \lambda_p T</math>;</p> <p><math>K_2^*</math>, base area/surface area for a drop;</p> <p><math>K_{21}</math>, <math>K_2^* + l_p \dot{Q}_{21}'' / \lambda_p T_1</math>;</p> <p><math>K_3</math>, a constant;</p> <p><math>l_p</math>, thickness of promoter layer;</p> <p><math>m</math>, constant in equation (1);</p> <p><math>N(r)</math>, distribution function; number of drops in size range <math>r, r + dr</math> divided by <math>dr</math>;</p> <p><math>n</math>, number of falling drops per (time · width) crossing a horizontal line on the surface;</p> <p><math>n_p</math>, number of adhering drops per area in size range <math>\hat{r}, \hat{r}_p</math>;</p> <p><math>\dot{Q}_b''</math>, heat-transfer rate through drop divided by base area;</p> <p><math>\dot{Q}_1''</math>, <math>\lambda_f T h_{fg} / 2\sigma v_f</math>;</p> <p><math>\dot{Q}_2''</math>, <math>\frac{h_{fg}^2}{v_g} \left( \frac{\gamma - 1}{\gamma + 1} \right) \left( \frac{2\pi}{RT} \right)^{1/2}</math>;</p> <p><math>\dot{Q}_{21}''</math>, <math>\dot{Q}_2''</math> evaluated at <math>T = T_1</math>;</p> <p><math>R</math>, specific ideal-gas constant;</p> <p><math>r</math>, radius of drop;</p> <p><math>\hat{r}</math>, minimum drop radius, <math>2\sigma v_f T / h_{fg} \Delta T</math>;</p> <p><math>\hat{r}</math>, effective maximum radius of adhering drop;</p> <p><math>r_{\max}</math>, radius of largest non-sliding drop;</p> <p><math>r_p</math>, mean radius of largest primary drop;</p> <p><math>\hat{r}_z</math>, value of <math>\hat{r}</math> at distance <math>z</math> from top of surface;</p> <p><math>T</math>, temperature (thermodynamic temperature) of vapour;</p>	<p><math>T_1</math>, vapour saturation temperature for pressure of 1 atm;</p> <p><math>V</math>, volume condensation rate per area;</p> <p><math>V_0</math>, value of <math>V</math> at <math>z = z_0</math>;</p> <p><math>\bar{V}_0</math>, average value of <math>V_z</math> in the range <math>0 &lt; z &lt; z_0</math>;</p> <p><math>V_z</math>, value of <math>V</math> at distance <math>z</math> from top of surface;</p> <p><math>v_z</math>, volume of a falling drop at distance <math>z</math> from top of surface;</p> <p><math>v_f</math>, specific volume of liquid phase;</p> <p><math>v_g</math>, specific volume of vapour phase;</p> <p><math>z</math>, distance from top of condensing surface;</p> <p><math>z_0</math>, distance from top of condensing surface above which falling drops originate.</p> <p style="text-align: center;">Greek symbols</p> <p><math>\alpha</math>, fraction of area covered by drops with radius greater than <math>r</math>;</p> <p><math>\gamma</math>, ratio of principal specific heat capacities (<math>c_p/c_v</math>) for vapour;</p> <p><math>\Delta T</math>, vapour-to-surface temperature difference;</p> <p><math>\lambda_f</math>, thermal conductivity of liquid phase;</p> <p><math>\lambda_p</math>, thermal conductivity of promoter layer;</p> <p><math>\mu</math>, number of condensation sites per area;</p> <p><math>\nu</math>, sweeping frequency;</p> <p><math>\nu_0</math>, sweeping frequency at <math>z = z_0</math>;</p> <p><math>\nu_z</math>, sweeping frequency at distance <math>z</math> from top of surface;</p> <p><math>\rho_f</math>, density of liquid phase;</p> <p><math>\rho_g</math>, density of vapour phase;</p> <p><math>\sigma</math>, liquid-vapour interfacial tension.</p>
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### INTRODUCTION AND OUTLINE OF THEORY

A THEORY of heat transfer by dropwise condensation [1], models the dropwise condensation process as a steady distribution of non-growing, non-coalescing drops. The heat-transfer rate through a single drop is calculated and the average heat flux found by integration over all drops. The equation used for the distribution of drop sizes was

$$\alpha = 1 - (r/\hat{r})^m \quad (1)$$

where  $m$  is a constant. Equation (1) has the property

that it gives zero area covered by drops greater than  $\hat{r}$  and complete coverage as the radius of the smallest drop tends to zero.

The equation for the mean heat flux through a single drop may be written

$$\dot{Q}_b'' = \frac{\Delta T - \frac{2\sigma v_f T}{rh_{fg}}}{K_1 \left(\frac{r}{\lambda_f}\right) + K_2^* \left(\frac{T}{\dot{Q}_2''}\right) + \left(\frac{l_p}{\lambda_p}\right)}. \quad (2)$$

The numerator is the available vapour-to-surface temperature difference less the amount by which the vapour must be sub-cooled below its normal "saturation" temperature in order that condensation may take place on the convex liquid surface. The denominator is an approximate representation of the combined resistances of the liquid, the vapour-liquid interface and the promoter layer. The constant  $K_1$  is associated with conduction in the liquid.  $K_2^*$  is the ratio of the base area to that of the curved surface of a drop. In the original theory [1], the last two terms in the denominator of equation (2) were combined so as to generate a promoter-dependent constant  $K_{21}$  whose value also depends on the drop shape. When the promoter resistance is negligible  $K_{21}$  is equal to  $K_2^*$ .

Using equations (1) and (2) and integrating between  $\hat{r}$  and  $\hat{r}$ , the total heat flux was found as a function of vapour-to-surface temperature difference.  $\hat{r}$ , the mean radius of the largest adherent drop, was estimated on grounds of dimensional analysis by

$$\hat{r} = K_3 \{\sigma/g(\rho_f - \rho_g)\}^{1/2}. \quad (3)$$

The constants  $K_1$ ,  $K_{21}$ ,  $K_3$  and  $m$  were known to be of order unity but their actual values were chosen so as best to fit the available heat-transfer data, i.e. heat flux-temperature difference measurements for steam at near-atmospheric pressures. This has led to criticism of the theory and will be discussed briefly before turning to the present extension to include the effect of plate height.

Firstly, it may be noted that the theory was subsequently found (when using the original values of the constants) to predict closely later measurements for lower steam pressures [2-5]. Secondly, values close to those actually chosen, for three of the four constants, may be obtained with recourse to the heat flux-temperature difference measurements. In the case of the remaining (promoter-dependent) constant, a lower limit may be set (for the case where the promoter itself offers negligible thermal resistance) without reference to heat-transfer measurements. The values actually chosen [1] varied between this lower limit and somewhat higher values according to promoter.

Equation (1) has received support from photographic measurements [2, 3, 6] and from a theoretical study of the drop size distribution [7] which makes no appeal either to the heat-transfer theory or measurements. Both indicate  $m \simeq 1/3$ . A crude model for conduction in the drop [8] gives for hemispherical drops  $K_1 \simeq 2/3$ . Visual observations for dropwise condensation of steam indicate  $\hat{r} \simeq 1$  mm, which gives  $K_3 \simeq 0.4$ . These

values of  $m$ ,  $K_1$  and  $K_3$  are, in fact, those adopted in [1]. The constant  $K_{21}$  depends on the promoter resistance† which, if negligible, gives  $K_{21} = K_2^*$ , i.e. the ratio of the base area to the area of the curved surface. Since, in the case of water the drops are almost hemispherical, the lower bound for  $K_{21}$  is about 1/2. The values adopted in [1] varied from promoter to promoter in the range 1/2 to about 2. Thus, had no attempt been made to include dependence of the heat transfer on promoter used, which is relatively small for "monolayer type" promoters, the theory would involve no constants whose values cannot be found independently of heat-transfer data.

A more fundamental objection to the theory is its steady treatment of a highly non-steady process. In the real case the surface temperature is both non-uniform and, at any location, varies with time. Additionally, in view of the high rate of coalescence between drops [9], it might be thought that the steady conduction calculation for each drop could be seriously in error.

Surface temperature variations arise from the fact that any given location plays different roles at different times. A small area is "bare" immediately after exposure caused by coalescence of a pair of drops, one of which, prior to coalescence, covered it. A short time later the same area will be covered with a large number of very small rapidly growing "primary" drops and somewhat later, following many coalescences, by a smaller number of larger drops. This process continues with time, i.e. the sizes of the largest drops increase and their number decreases while smaller drops appear in the spaces between them, until the small area in question is either completely covered by one of its largest drops, or either covered or again rendered bare, by coalescence between larger neighbouring drops. A totally covered region will, at some later time, again become exposed due to movement of the covering drop. With a much lower frequency than that of the events described above, relatively large areas are swept from time to time by falling drops.

The role played by any given location varies with great rapidity due to the close packing of drops of all sizes and consequent rapid re-distribution of condensate resulting from the high rate of coalescence. It would seem that the temperature of a small region of the surface would depend on the role it played at the time in question. In this case the surface temperature at any instant would be non-uniform and the temperature at any position would vary with time.

The fact that the steady theory is apparently successful and that heat-transfer measurements do not depend significantly on the material of the condensing surface [10]‡ suggests that the temperature fluctuations and

†  $l_p/\lambda_p$  might represent any resistance associated with the promoter i.e. not necessarily due to conduction in a thin layer.

‡ A theoretical analysis of the effect of the thermal properties of the surface has been given [11] and a recent report [12] has indicated that overall coefficients for dropwise condensation on tubes of different materials differed signifi-

non-uniformities are insignificant. It is possible that the frequency of the fluctuations associated with growth and coalescence is so high that the surface temperature at any position is essentially constant and hence uniform for the whole surface, while at the same time, the intermittent disturbances associated with "sweeping" occur with such low frequency as to have insignificant effect.

Regarding the steady treatment of heat transfer through a single drop, it would seem that, following a coalescence, the temperature distribution in the newly-formed drop might differ radically from that associated with steady conduction, particularly after coalescence between drops of roughly equal size and in view of the fact that the time interval between successive coalescences is very short. However, the time interval must be considered in relation to that during which two drops, upon contact, coalesce and the associated disturbances die away. That the time for coalescence is much shorter than that during which drops grow without coalescing, is illustrated by the fact that still photographs, showing several hundreds of drops, rarely (for a well-promoted surface) show drops in the process of coalescence, and also by the fact that successive frames of cine films show drops before and after coalescence.

In the present work, it is first demonstrated that the theory [1] accurately predicts the effect of maximum drop size on the heat-transfer coefficient. Thus for a given vapour-to-surface temperature difference, the relationship between local maximum drop size and volume condensation rate is known. This relationship is used, together with certain simplifying assumptions and an equation relating local maximum drop size, local mean volume condensation rate per area and local sweeping frequency, based on the drop growth model [7], to determine the height-dependence of the sweeping frequency and hence of the heat-transfer coefficient.

Finally it is shown that equation (1) can be used to estimate nucleation site densities.

#### THE EFFECTIVE MAXIMUM DROP SIZE

There is some doubt as to the precise meaning and hence the correct value of  $\hat{r}$  which occurs in equation (1) and as the upper limit of the integral giving the total heat transfer. It has recently been pointed out [7] that drops grow in distinct "generations". The largest drops, at a given location and at a given instant, are all of much the same size, and constitute the initial generation. In the spaces between these largest drops are significantly smaller drops, also of roughly equal size, of the next generation. A third generation of drops exists in the spaces between those of the second generation and so on. Generations of drops may be clearly

identified in cine-films [9] and in a computer simulation [13].

The "effective maximum" drop radius  $\hat{r}$  at a given location is thus seen to be the average radius of the largest generation immediately prior to sweeping. Using the computer simulation [7] of the growth of a generation of drops, the radius of the largest drop of a generation is found to be around 1.3 times the mean radius. Hence  $\hat{r}$  is about  $r_{\max}/1.3$  where  $r_{\max}$  is the radius of the largest drop at the end of the growth cycle.

Measurements have recently been made of the dependence of the steam-side heat-transfer coefficient on the maximum drop size [14–16]. These results are shown in Fig. 1. The theory shows (see equation (30) of [1]) that the heat flux, and hence the heat-transfer coefficient for a given vapour-to-surface temperature difference, varies as  $\hat{r}^{-1/3}$ . The theoretical dependence† of the heat-transfer coefficient on the effective maximum drop size (estimated as  $r_{\max}/1.3$ ) is shown in Fig. 1.

#### FALLING DROPS AND SWEEPING FREQUENCY

Falling drops play a major role in determining the heat transfer, by sweeping vertical tracks clear of adherent drops. Also, being much larger than the adherent drops, the falling drops have virtually infinite heat-transfer resistance and effectively "blanket" part of the surface. Since their growth mechanism (by sweeping up adherent drops) is different, the falling drops are not included in the size distribution function for adherent drops.

The dropwise condensation "cycle" comprises two parts: the "growth time" during which drops nucleate (on a newly-swept region), grow and coalesce; and the "sweeping time" during which the region remains covered by the falling drop. For any horizontal strip of the condensing surface, the fractional coverage by falling drops is, on average, equal to the ratio of the sweeping time to the total cycle period. The fact that the proportion of the total condensing area covered by falling drops is generally small [17] (only for extremely high condensation rates or for near-horizontal surfaces is this appreciable) indicates that the growth time is in general much larger than the sweeping time and that the "sweeping frequency" (reciprocal of total cycle period) in general approximates to the reciprocal of the growth time.

The sweeping frequency, maximum drop size and heat-transfer coefficient depend on distance down the surface, surface inclination and vapour velocity. However, for constant  $\Delta T$  the quantities  $v$ ,  $\hat{r}$  and  $h$  are related. Thus

$$h \propto \hat{r}^{-1/3} \quad (4)$$

as indicated by [1] and discussed above, and, as is shown later,

$$\hat{r} \propto Vv^{-1}. \quad (5)$$

Footnote continued from p. 1364.

cantly. Since these tubes were not given identical surface coatings, it may be that the observed variations between tubes were due to differences in promoter effectiveness on the different surfaces rather than to the thermal properties of the materials.

†Taking  $\Delta T = 2.3$  K (the mean of the experimental values [14–16]) and  $K_{21} = 2.155$  (for the promoter oleic acid [1]). The predicted values of  $h$  vary only slightly in the experimental range of  $\Delta T$  (1.6–3.3 K). The slope of the theoretical line depends neither on  $\Delta T$  nor  $K_{21}$ .

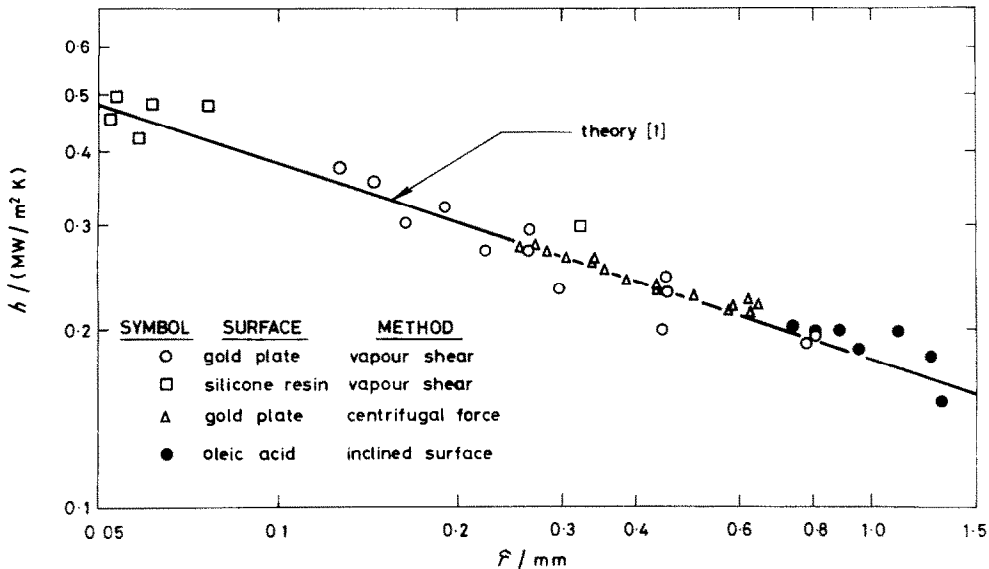


FIG. 1. Dependence of heat-transfer coefficient on maximum drop size. Comparison of measurements [14–16] and theory [1].

Hence, for constant  $\Delta T$ , when  $h \propto V$ ,

$$h \propto v^{1/4}. \quad (6)$$

In (5) and (6)  $v$  is strictly the reciprocal of the growth time and should be so taken for those extreme cases where coverage by falling drops is appreciable.

#### EFFECT OF PLATE HEIGHT

It might at first be thought that, since all of the condensate from upper parts of the surface must drain via the lower parts, the heat-transfer coefficient would decrease with distance down the plate. That this is not the case [18] is due to three factors:

- Coverage by falling drops is in general small, and variations with height have correspondingly small effect on the heat-transfer coefficient.
- Increased coverage due to growth of falling drops as they proceed down the surface is counteracted by the fact that the falling drops accelerate and consequently become more widely spaced. Alternatively, a falling drop may, despite its increase in size, cover a lower region for a shorter time than it does a higher one.
- Since falling drops grow as they proceed down the surface, they sweep diverging tracks. Consequently, lower regions are swept more frequently, and hence, from (6), the heat-transfer coefficient *increases* with distance down the surface.

With the aid of certain simplifying assumptions, and for fixed vapour and surface temperatures, the variation, with distance down the surface, resulting from (c) above, of the sweeping frequency and hence of the heat-transfer coefficient, may be calculated.

Since the sweeping frequency increases with distance down the surface, only drops near to the top are able to reach the size at which they begin to fall. We shall assume that all falling drops originate within some short distance  $z_0$  of the top of the surface. Thus for

$z > z_0$  the number of drops per (time  $\times$  width),  $n$ , crossing any horizontal line on the surface is independent of  $z$ .

The local sweeping frequency,  $v_z$ , is given by

$$v_z = nb_z \quad (7)$$

where  $b_z$  is the local width of the swept track. We assume also that a falling drop retains its shape as it proceeds down the surface, i.e. falling drops have similar shapes at different heights. (This may be invalid for the lower parts of tall surfaces, when a falling drop may extend lengthwise into a streak whose width remains roughly constant. In this case the sweeping frequency, effective maximum drop size and heat-transfer coefficient would, in the absence of appreciable coverage by falling drops, remain constant with height.)

With the above assumption, the volume of a falling drop,  $v$ , may be written in terms of its maximum width  $b_z$ :

$$v = c_1 b_z^3 \quad (8)$$

where  $c_1$  is a constant shape factor for falling drops. For hemispherical falling drops  $c_1 = \pi/12$ . In the steady state, the total volume of condensate in the form of moving drops, crossing a horizontal line at distance  $z$  from the top of the surface, is equal to the volume condensation rate on the region above, i.e.

$$nv_z = \int_0^z V_z dz \quad (9)$$

where  $V_z$  is the local condensation rate per area.

Thus from (8) and (9):

$$b_z^3 = \frac{1}{c_1 n} \int_0^z V_z dz. \quad (10)$$

Since we are considering a fixed value of  $\Delta T$ , it follows from (4) that

$$V_z = c_2 \hat{r}_z^{-1/3} \quad (11)$$

where  $c_2$  is independent of  $z$ .

Differentiating (10) and substituting for  $V_z$  from (11):

$$3b_z^2 \frac{db_z}{dz} = \left(\frac{c_2}{c_1 n}\right) \hat{r}_z^{-1/3}. \tag{12}$$

On dimensional grounds one might expect:

$$\hat{r}_z = c_3 V_z v_z^{-1} \tag{13}$$

where  $c_3$  is a constant.

Moreover equation (13) may also be obtained (see Appendices A and B) by using the drop growth model [7] to calculate the average volume of condensate on a given area immediately prior to sweeping, which, when multiplied by the local sweeping frequency, gives the local volume condensation rate. This calculation gives  $c_3 = 2.6$ . From (7), (11) and (13):

$$\hat{r}_z = \left(\frac{c_2 c_3}{n b_z}\right)^{3/4}. \tag{14}$$

Substituting for  $\hat{r}_z$  from (14) in (12) and integrating yields:

$$b_z^{11/4} - b_0^{11/4} = \frac{1}{12} \frac{1}{c_1 c_3^{1/4}} \left(\frac{c_2}{n}\right)^{3/4} (z - z_0) \tag{15}$$

where  $b_0$  is the width of the swept track at  $z = z_0$ .

From (7) and (15) we have the relationship between sweeping frequency and distance from the top of the condensing surface:

$$v_z^{11/4} - v_0^{11/4} = \frac{1}{12} \frac{n^2}{c_1 c_3^{1/4}} c_2^{3/4} (z - z_0) \tag{16}$$

where  $v_z = v_0$  at  $z = z_0$ .

From (14) and (15) we have the relationship between the effective maximum drop size and distance from the top of the condensing surface:

$$\hat{r}_z^{-11/3} - \hat{r}_0^{-11/3} = \frac{1}{12} \frac{n^2}{c_1 c_2^2 c_3^3} (z - z_0). \tag{17}$$

From (11) and (17) we obtain the height-dependence of the volume condensation rate:

$$V_z^{11} - V_0^{11} = \frac{1}{12} \frac{n^2 c_2^9}{c_1 c_3^3} (z - z_0). \tag{18}$$

Since, for constant  $\Delta T$ , the local heat-transfer coefficient is proportional to  $V_z$ , its dependence on  $z$  takes the form

$$h_z = \{A(z - z_0) + B\}^{1/11} \tag{19}$$

where  $A$  and  $B$  are constants.

COMPARISON WITH MEASUREMENTS

Unfortunately, few data are available with which the above may be compared.

Sweeping frequency

The only measurements known to the author are those of Sugawara and Michiyoshi [19], who observed the sweeping frequency at three different heights on a vertical surface, for two different coolant flow rates. When these data are fitted by an equation of the form

$$v_z^m = \phi z + \psi \tag{20}$$

where  $\phi$  and  $\psi$  were allowed to take different values for the two data sets,  $m$  was found to be 2.79, i.e. very close to the theoretical value of 11/4 [see equation (16)].

Equation (16) may be simplified using the values of the constants found in Appendix A to give:

$$v_z^{11/4} = \left(\frac{V_0}{\hat{r}_0}\right)^{11/4} \left\{ 4.67 \left(\frac{V_z}{V_0}\right)^{3/4} \left(\frac{\hat{r}_z}{\hat{r}_0}\right)^{1/4} \left(\frac{z - z_0}{\hat{r}_0}\right) + 13.9 \right\} \tag{21}$$

from (17) and (18)

$$\left(\frac{\hat{r}_z}{\hat{r}_0}\right)^{1/4} \simeq 1 \quad \text{and} \quad \left(\frac{V_z}{V_0}\right)^{3/4} \simeq 1.$$

Using these approximations and the value of  $z_0$  from Appendix A, we obtain:

$$v_z = \frac{V_0}{\hat{r}_0} \left\{ 4.67 \left(\frac{z}{\hat{r}_0}\right) + 1.2 \right\}^{4/11} \tag{22}$$

and using  $\hat{r}_0 \simeq 1$  mm,

$$v_z = \left(\frac{V_0}{\text{mm}}\right) \left\{ 4.67 \left(\frac{z}{\text{mm}}\right) + 1.2 \right\}^{4/11}. \tag{23}$$

Equation (23), with values of  $V_0$  found by least-squares fitting, is compared with the measurements in Fig. 2. Unfortunately the authors [19] do not give the condensation rates for comparison with the values found for  $V_0$ .

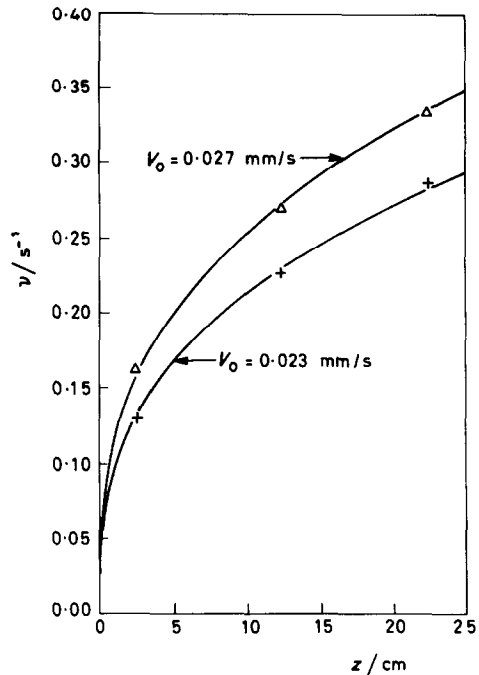


FIG. 2. Dependence of sweeping frequency on distance from top of condensing surface. Comparison of measurements [19] with equation (23).

Heat-transfer measurements

Le Freve and Rose [18] made measurements at distances from the top of the condensing surface of 25, 28 and 102 mm and observed no height-dependence. Wenzel [20] made measurements at four different heights on a plate of total height 360 mm and did not

report differences between the results obtained at different locations. These results are in accord with the extremely weak dependence of heat-transfer coefficient on height indicated by equation (19).

#### NUCLEATION SITE DENSITY

A continuous distribution of drop sizes, such as that given by equation (1), cannot be used to calculate the number of drops of a given size but rather, the number of drops in a given size range. Thus equation (1) does not directly provide a value for the density of nucleation sites. However, if we can estimate the radius  $r_p$  to which primary drops† grow, on average, before coalescing, then equation (1) can be used to determine the number of primary drops, i.e. drops in the size range  $\check{r}$ ,  $r_p$  as well as the area available for these drops, i.e. the total area less that covered by all drops larger than  $r_p$ . Thus we may obtain the number of primary drops per area available for them which, since each drop represents a nucleation site, gives the nucleation site density. For hemispherical drops equation (1), with  $m = 1/3$ , gives

$$N(r) = \frac{1}{3\pi r^2 \check{r}} \left(\frac{r}{\check{r}}\right)^{-2/3} \quad (24)$$

The number of primary drops per area of condensing surface  $n_p$  is thus given by:

$$n_p = \int_{\check{r}}^{r_p} N(r) dr = \frac{\check{r}^{-5/3} - r_p^{-5/3}}{5\pi \check{r}^{1/3}} \quad (25)$$

The fraction of the total area available for these drops is, from (1), equal to  $(r_p/\check{r})^{1/3}$ . Hence, the nucleation site density is given by:

$$\mu = \frac{\check{r}^{-5/3} - r_p^{-5/3}}{5\pi r_p^{1/3}} \quad (26)$$

$r_p$  may be estimated as half the distance between neighbouring sites in a uniform triangular array. In the real case, primary drops will reach a radius larger than this estimate, before coalescing, in regions where sites have greater than average spacing and will undergo coalescence at a smaller radius where the sites are more closely packed than average. We thus estimate:

$$r_p = \{(2\sqrt{3})\mu\}^{-1/2} \quad (27)$$

From (26) and (27) we obtain

$$\mu = 0.037/\check{r}^2 \quad (28)$$

Estimates of nucleation site density from optical microscope photographs [2, 3] and from electron microscope photographs [14–16] have recently been obtained. Griffith and Graham [2, 3] give values of  $2 \times 10^8 \text{ cm}^{-2}$  and  $6 \times 10^8 \text{ cm}^{-2}$  for conditions for which  $\check{r} \approx 0.07 \mu\text{m}$ . Using this value of  $\check{r}$ , (28) indicates a site density of  $7.5 \times 10^8 \text{ cm}^{-2}$ . Tanasawa and co-workers found values exceeding  $10^{10} \text{ cm}^{-2}$  for conditions under

which the radius of the smallest drops was around  $0.01 \mu\text{m}$ . For  $\check{r} = 0.01 \mu\text{m}$ , (28) gives  $\mu = 3.7 \times 10^{10} \text{ cm}^{-2}$ . The calculated and observed site densities are seen to be in good agreement, particularly when bearing in mind the uncertainty in the values of  $\check{r}$ ; for instance, if the true value of  $\Delta T$  were as little as  $0.1 \text{ K}$  lower than that reported [2, 3], then the value of  $\check{r}$  would be in excess of  $0.1 \mu\text{m}$  and the calculated nucleation site density about  $3 \times 10^8 \text{ cm}^{-2}$ . It is of interest to note that the above site densities are within a factor of about 10 of the maximum possible values, i.e. for a uniform close-packed triangular array. For  $\check{r}$  having values of  $0.07 \mu\text{m}$  and  $0.01 \mu\text{m}$  the maximum site densities are  $5.9 \times 10^9 \text{ cm}^{-2}$  and  $2.9 \times 10^{11} \text{ cm}^{-2}$  respectively.

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†Drops which first form on bare surface and which have not undergone coalescence.

## APPENDIX A

Consideration of the Constants  $c_1, c_2, c_3, n$  and  $z_0$

- $c_1$ : If falling drops are taken to be hemispherical then  $c_1 = \pi/12$ .
- $c_2$ : Observation and theory (see Fig. 1) indicate that, for a given vapour-to-surface temperature difference and for a particular promoter, the product of the volume condensation rate per area and the cube root of the effective maximum drop size is constant. Thus, while  $V_z$  and  $\hat{r}_z$  both vary with distance down the condensing surface, the product  $c_2 = V_z \hat{r}_z^{1/3}$  is not dependent on  $z$ . For given values of  $\Delta T$  and  $K_{2,1}$ ,  $c_2$  may be obtained from equation (30) of [1] or from experimental values of  $V_z$  and  $\hat{r}_z$ , if available.
- $c_3$ : The drop growth model [7] can be used (see Appendix B) to show that the volume of adherent condensate per area of surface, immediately prior to sweeping, is equal to  $0.384\hat{r}_z$ , and hence

$$V_z = 0.384\hat{r}_z v_z. \quad (\text{A1})$$

Comparing (A1) with (13) gives  $c_3 = 2.6$ .

- $n$ : For  $z < z_0$ , drops are able to reach their maximum radius  $\hat{r}_0$  before beginning to slide. Since  $z_0$  is small, the radius of a falling drop at  $z_0$  is essentially equal to  $\hat{r}_0$ , thus, from (7):

$$v_0 = 2n\hat{r}_0 \quad (\text{A2})$$

and, from (A1):

$$V_0 = 0.384\hat{r}_0 v_0. \quad (\text{A3})$$

Then, from (A2) and (A3):

$$n = 1.3 V_0 / \hat{r}_0^2. \quad (\text{A4})$$

- $z_0$ : With the assumptions that falling drops at  $z_0$  have radius  $\hat{r}_0$  and are hemispherical, a matter balance for the region above  $z_0$  gives

$$n(\frac{2}{3}\pi\hat{r}_0^3) = \bar{V}_0 z_0 \quad (\text{A5})$$

where  $\bar{V}_0$  is the mean volume condensation rate per area for the region above  $z_0$ . Thus, from (A4) and (A5):

$$\left(\frac{z_0}{\hat{r}_0}\right) = 2.72 \left(\frac{V_0}{\bar{V}_0}\right) \quad (\text{A6})$$

and since  $z_0$  is small  $V_0 \simeq \bar{V}_0$  and

$$z_0 \simeq 2.72\hat{r}_0. \quad (\text{A7})$$

## APPENDIX B

Relation Between Local Sweeping Frequency, Maximum Drop Size and Condensation Rate

The total volume of condensate on a small region immediately prior to sweeping may be found by summing the volumes of all "generations" of drops [7] at this instant. Drops of generation 0 have their maximum radius  $\hat{r}_z$  and, for unit area, their total volume  $w_0$ , is given by

$$w_0 = \frac{2}{3} f \hat{r}_z \quad (\text{B1})$$

where the drops have been assumed hemispherical and where  $f$  is the fraction of area covered, which is shown [7] to be equal to about 0.55.

If successive generations are assumed to have their mean radius (all radii are equally probable), then, for unit total area, the volume of all drops of generation  $i$  ( $i > 0$ ) is given by

$$w_i = \frac{1}{3} f \hat{r}_z \gamma^i (1-f)^i \quad (\text{B2})$$

where  $\gamma$  is the ratio of the maximum radius of generation  $k+1$  to the radius at a given instant of generation  $k$ . The value of  $\gamma$  was found [7] to be 0.189. Thus, the total volume of condensate on unit area immediately prior to sweeping  $w_i$ , is given by

$$w_i = \frac{2}{3} f \hat{r}_z + \frac{1}{3} f \hat{r}_z \sum_{i=1}^m \gamma^i (1-f)^i \quad (\text{B3})$$

where  $m$  is the total number of generations. The series in (B3) converges so rapidly that the later generations contribute negligibly to the total volume and the sum to infinity can be used for  $w_i$ , thus

$$w_i = \frac{f \hat{r}_z}{3} \left\{ \frac{2 - \gamma(1-f)}{1 - \gamma(1-f)} \right\} \quad (\text{B4})$$

and using  $f = 0.55$ ,  $\gamma = 0.189$ ,

$$w_i = 0.384 \hat{r}_z. \quad (\text{B5})$$

The local volume condensation rate per area  $V_z$  is then given by

$$V_z = 0.384 \hat{r}_z v_z. \quad (\text{B6})$$

## NOUVEAUX ASPECTS DE LA THEORIE DE LA CONDENSATION EN GOUTTES

**Résumé**—La relation théorique qui lie le coefficient de transfert de chaleur à la dimension maximale des gouttes [1] est introduite dans une nouvelle méthode de calcul qui fournit la fréquence de balayage et le coefficient de transfert de chaleur en fonction de la distance sur la surface de condensation. Les résultats sont en accord satisfaisant avec les observations antérieures. On montre que l'influence de la hauteur sur le coefficient de transfert de chaleur est extrêmement faible. On montre également que la densité des points de nucléation peut être déduite d'une équation pour la distribution des dimensions des gouttes [1] et que les valeurs obtenues sont en accord satisfaisant avec les mesures aux microscopes optique et électronique.

## WEITERE ASPEKTE DER THEORIE ZUR TROPFENKONDENSATION

**Zusammenfassung**—Die theoretische Abhängigkeit des Wärmeübergangskoeffizienten von der maximalen Tropfengröße [1] wird in ein neues Verfahren zur Berechnung der Abhängigkeit der Abrollfrequenz und des Wärmeübergangskoeffizienten von der Lauflänge auf der Kondensationsfläche eingeführt. Beide Ergebnisse stimmen in befriedigender Weise mit früheren Beobachtungen überein. Die Abhängigkeit des Wärmeübergangskoeffizienten von der Lauflänge erweist sich als extrem gering. Es wird außerdem gezeigt, daß die Keimstellendichte aus einer Gleichung für die Verteilung der Tropfengröße abgeleitet werden kann [1] und daß die so erhaltenen Werte gut mit den mit Licht- und Elektronenmikroskopen ermittelten Meßwerten übereinstimmen.

## ДАЛЬНЕЙШЕЕ РАЗВИТИЕ ТЕОРИИ КАПЕЛЬНОЙ КОНДЕНСАЦИИ

**Аннотация** — В новом расчетном методе представлена теоретическая зависимость коэффициента теплообмена от максимального размера капли [1], показывающая зависимость частоты сноса и коэффициента теплообмена от расстояния вниз по поверхности конденсации. Оба результата удовлетворительно согласуются с более ранними данными. Показано, что зависимость коэффициента теплообмена от высоты капли крайне мала. Показано также, что плотность центров образования пузырьков выводится из уравнения распределения размеров капли [1] и что полученные значения удовлетворительно согласуются с данными измерений на оптическом и электронном микроскопах.